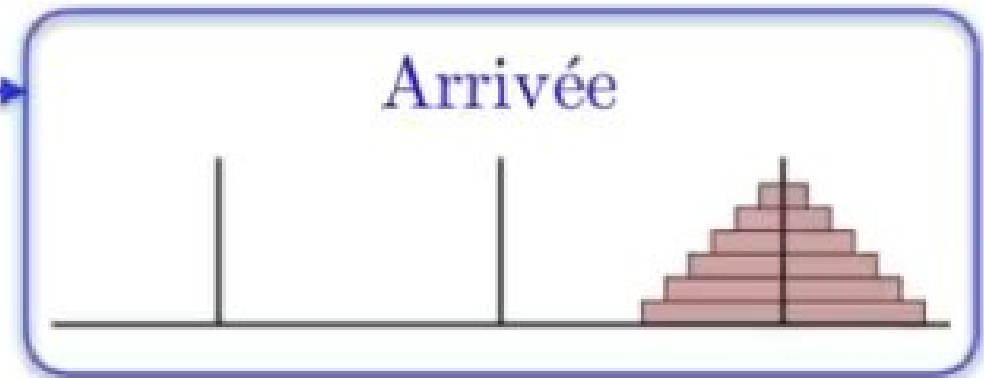
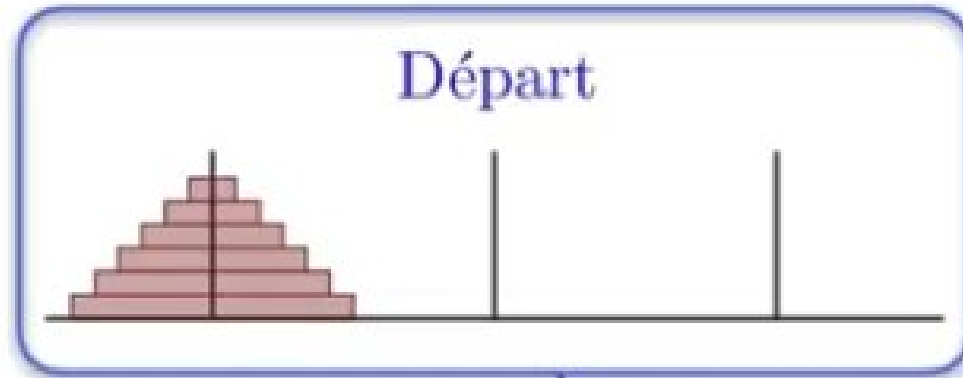


A close-up photograph of a person's hand interacting with a wooden Hanoi Tower puzzle. The hand is positioned to move a disc from the leftmost wooden peg to the middle one. The puzzle consists of three vertical wooden pegs and five wooden discs of varying sizes. The background is a blurred wooden surface.

TOUR DE HANOI

La tour de Hanoï



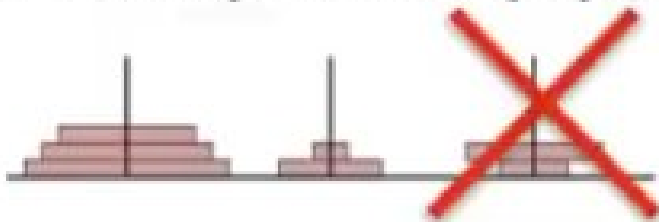
La tour de Hanoï

Départ



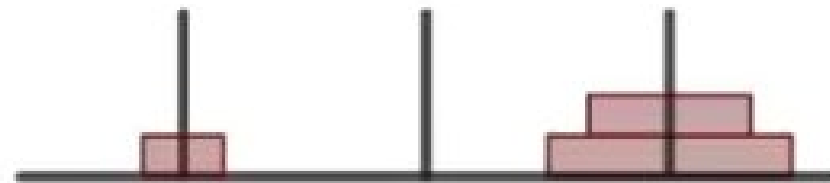
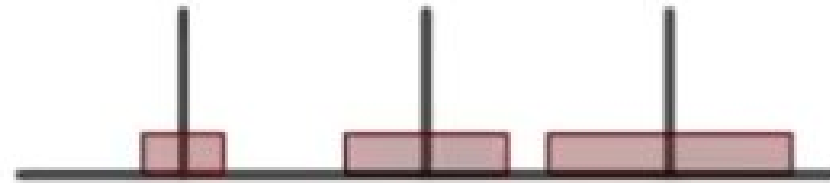
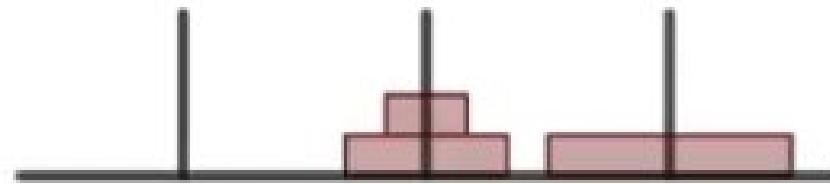
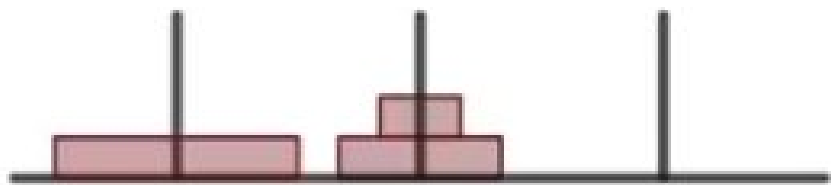
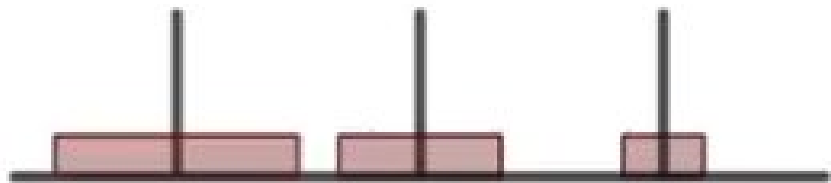
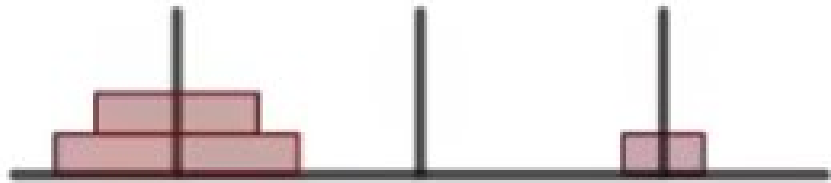
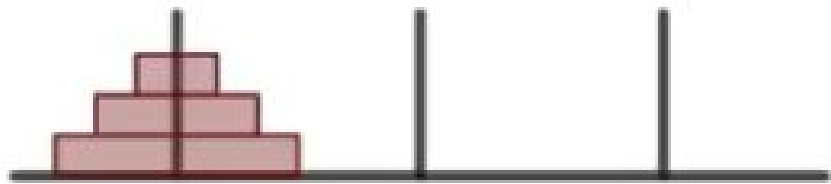
Règles

- On déplace un seul disque à la fois
- Jamais un disque sur un disque plus petit



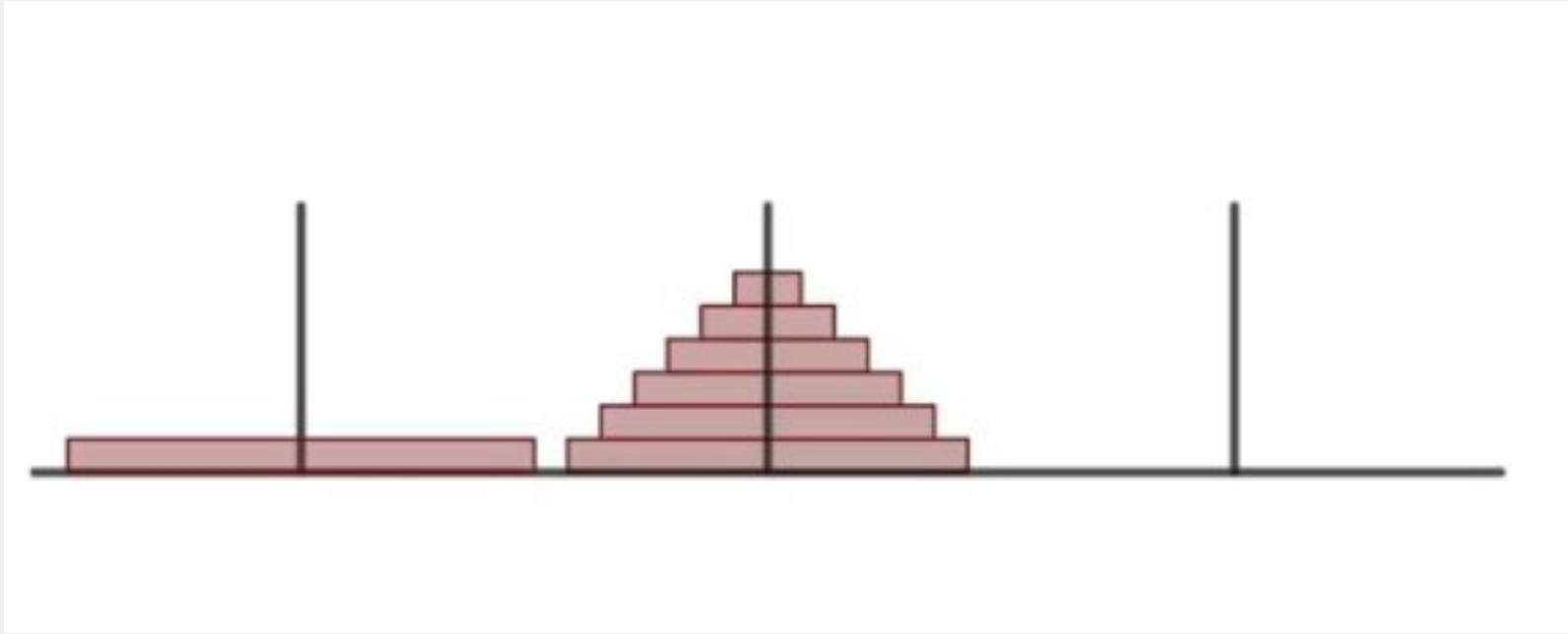
Arrivée

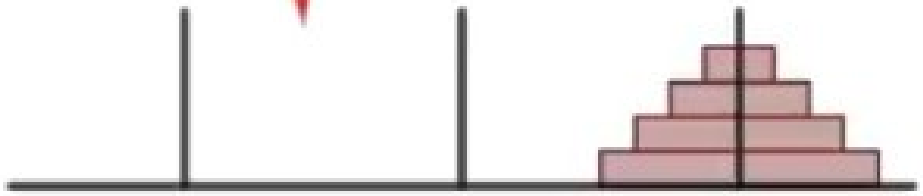
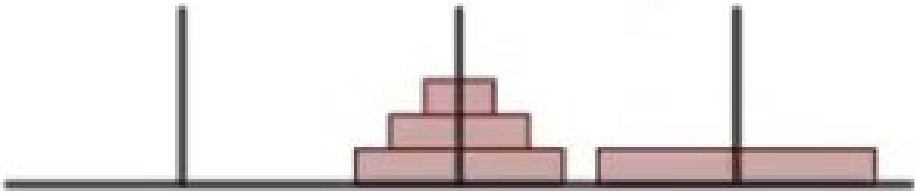
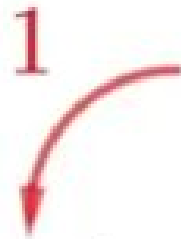
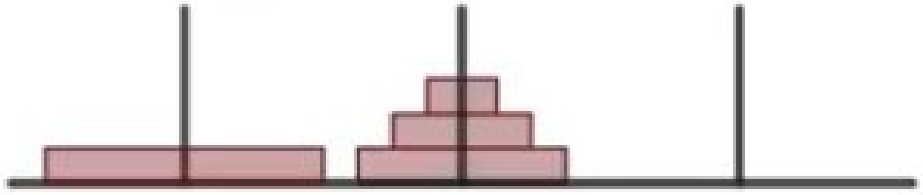
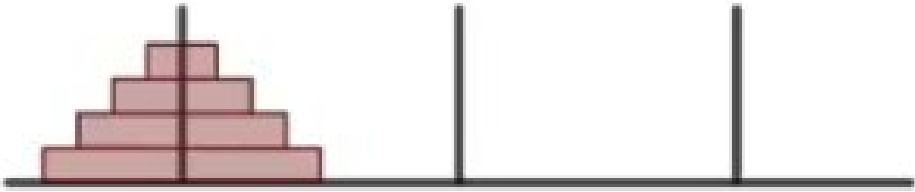


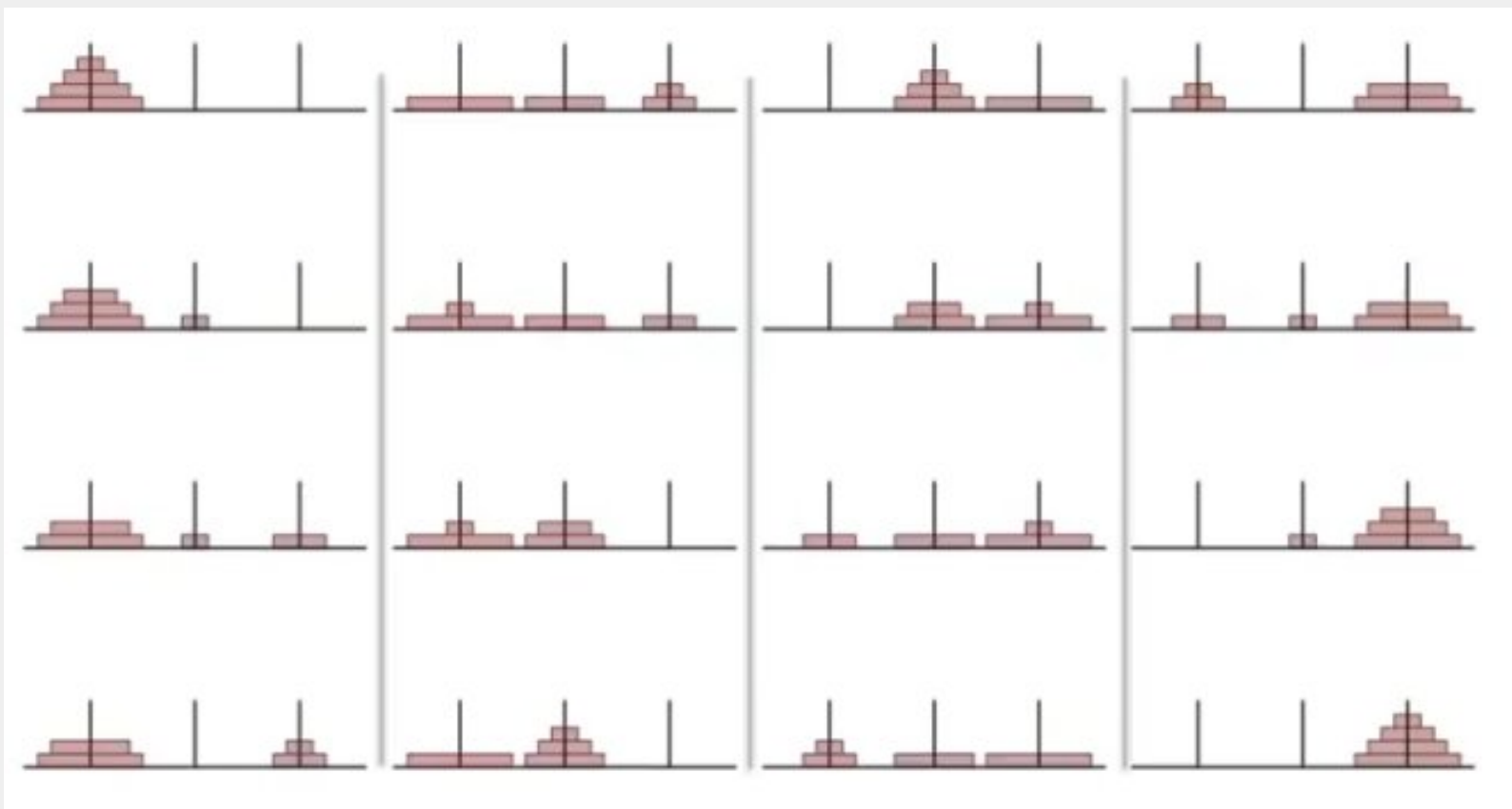




Édouard Lucas
(1842-1891)

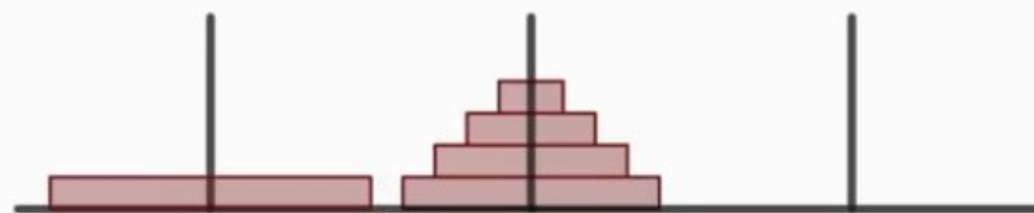




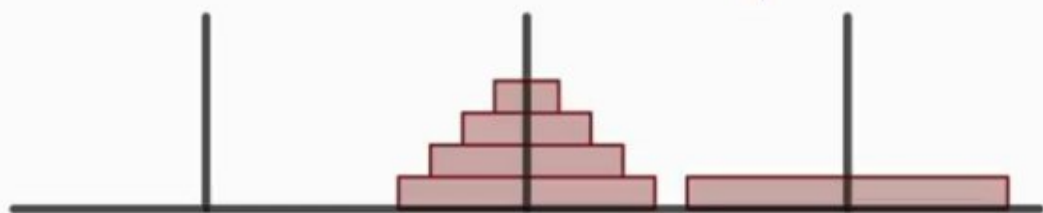




15



1



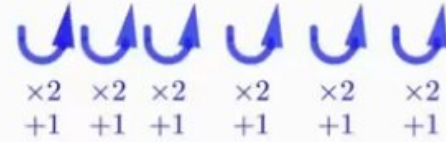
15



Nombre de disques	1	2	3	4	5	6	...	n
Nombre de déplacements	1	3	7	15	31	63	...	$2^n - 1$



Nombre de disques	1	2	3	4	5	6	...	n
Nombre de déplacements	1	3	7	15	31	63	...	$2^n - 1$



$$n = 64$$

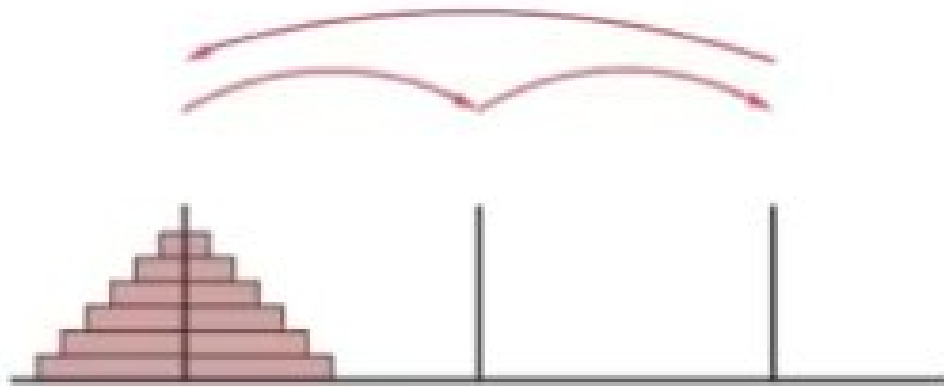
$$2^{64} - 1 \text{ déplacements}$$

$$= 18\,446\,744\,073\,709\,551\,615$$

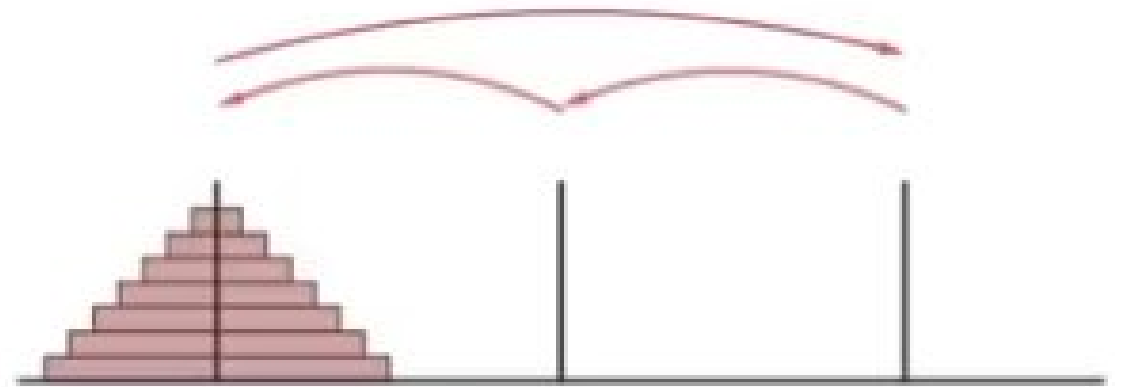


Le petit disque tourne toujours dans le même sens

Pour un nombre pair de disques :



Pour un nombre impair de disques :



Algorithme
récuratif

$\text{Hanoi}(n, X, Y)$

Si $n = 1$:

Déplacer le disque de X à Y ;

Sinon, faire :

$\text{Hanoi}(n - 1, X, Z)$;

Déplacer le disque n de X à Y ;

$\text{Hanoi}(n - 1, Z, Y)$.

Algorithme
itératif

Hanoi(n)

On appelle d le plus petit disque ;

Déplacer* d ;

Tant qu'un autre disque que d peut être déplacé :

 Déplacer cet autre disque ;

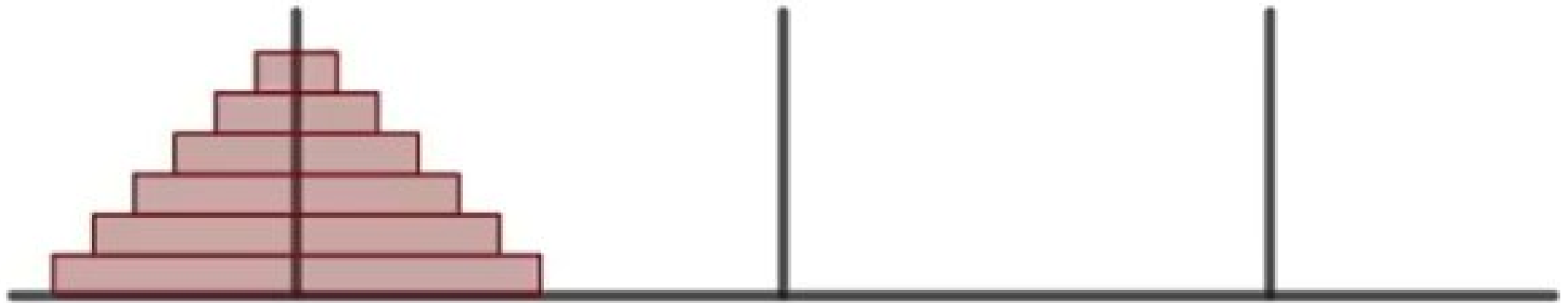
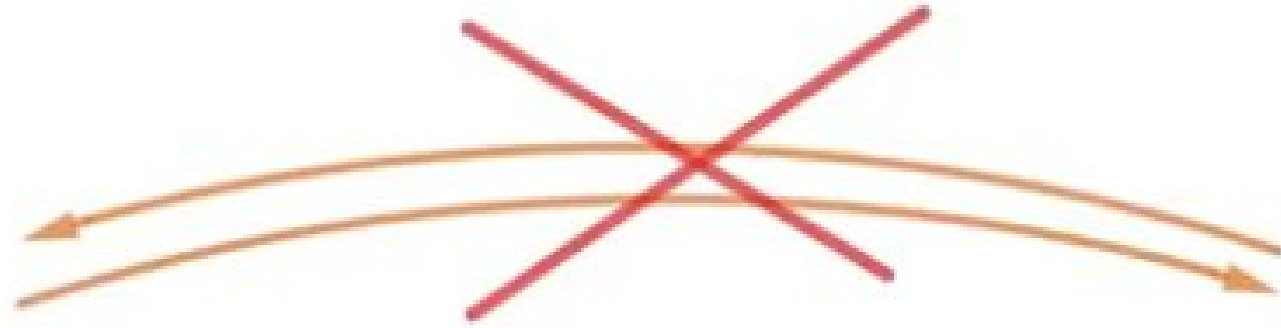
 Déplacer* d ;

* (dans le sens donné par la parité de n)

Nombre total d'états possibles avec n disques :

$$\overbrace{3 \times 3 \times 3 \times 3 \times \cdots \times 3}^n = 3^n$$

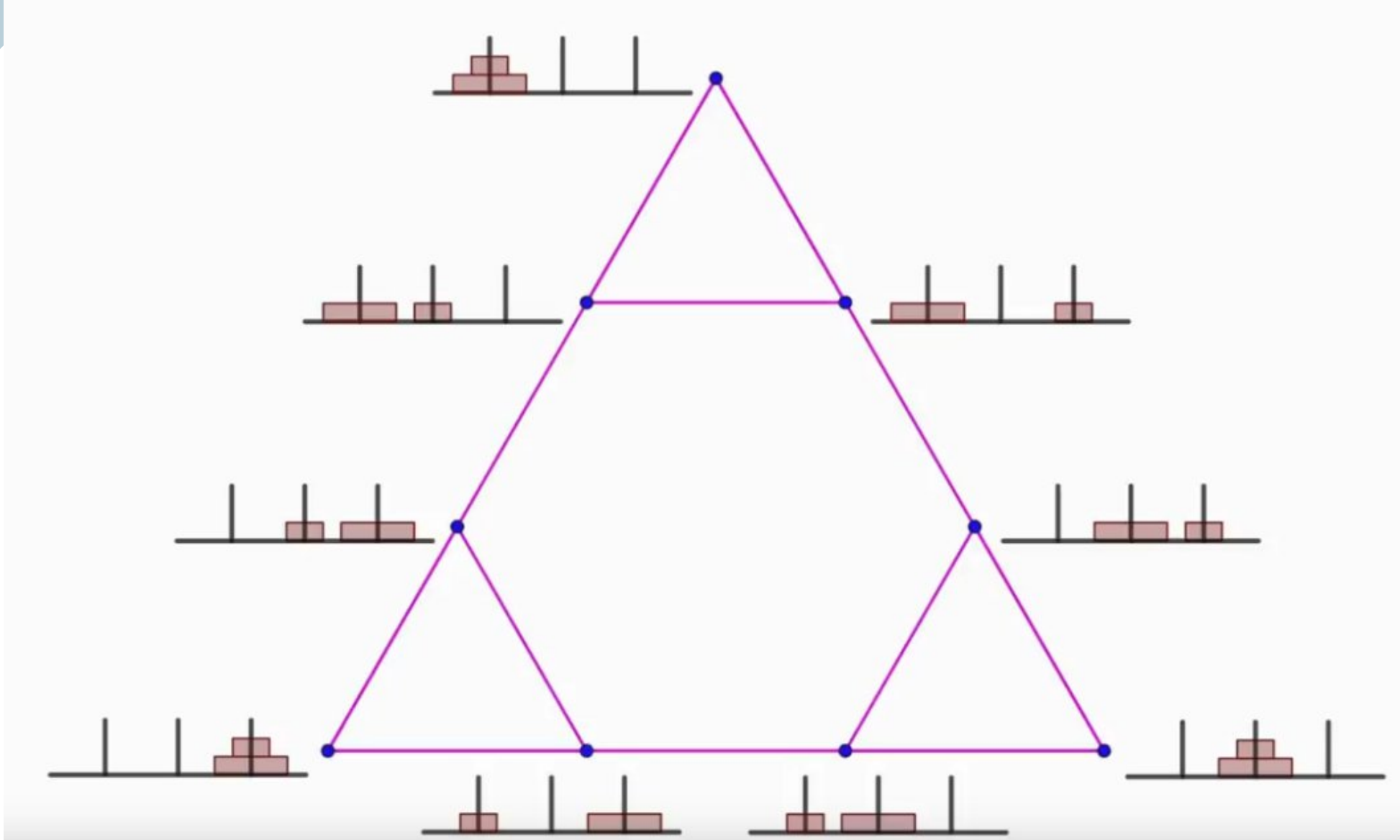
disque n disque $n-1$ disque $n-2$ disque $n-3$... disque 1

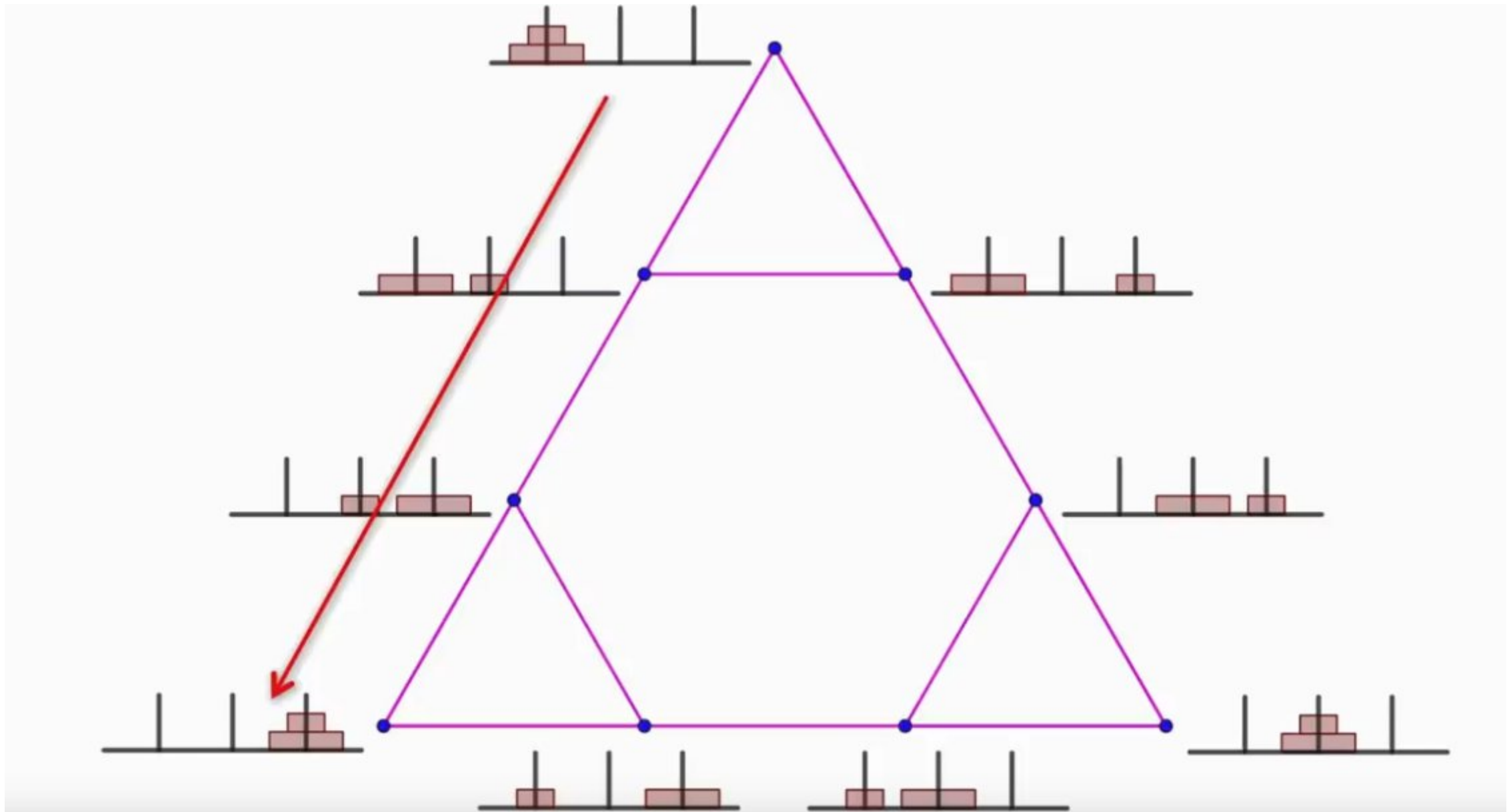


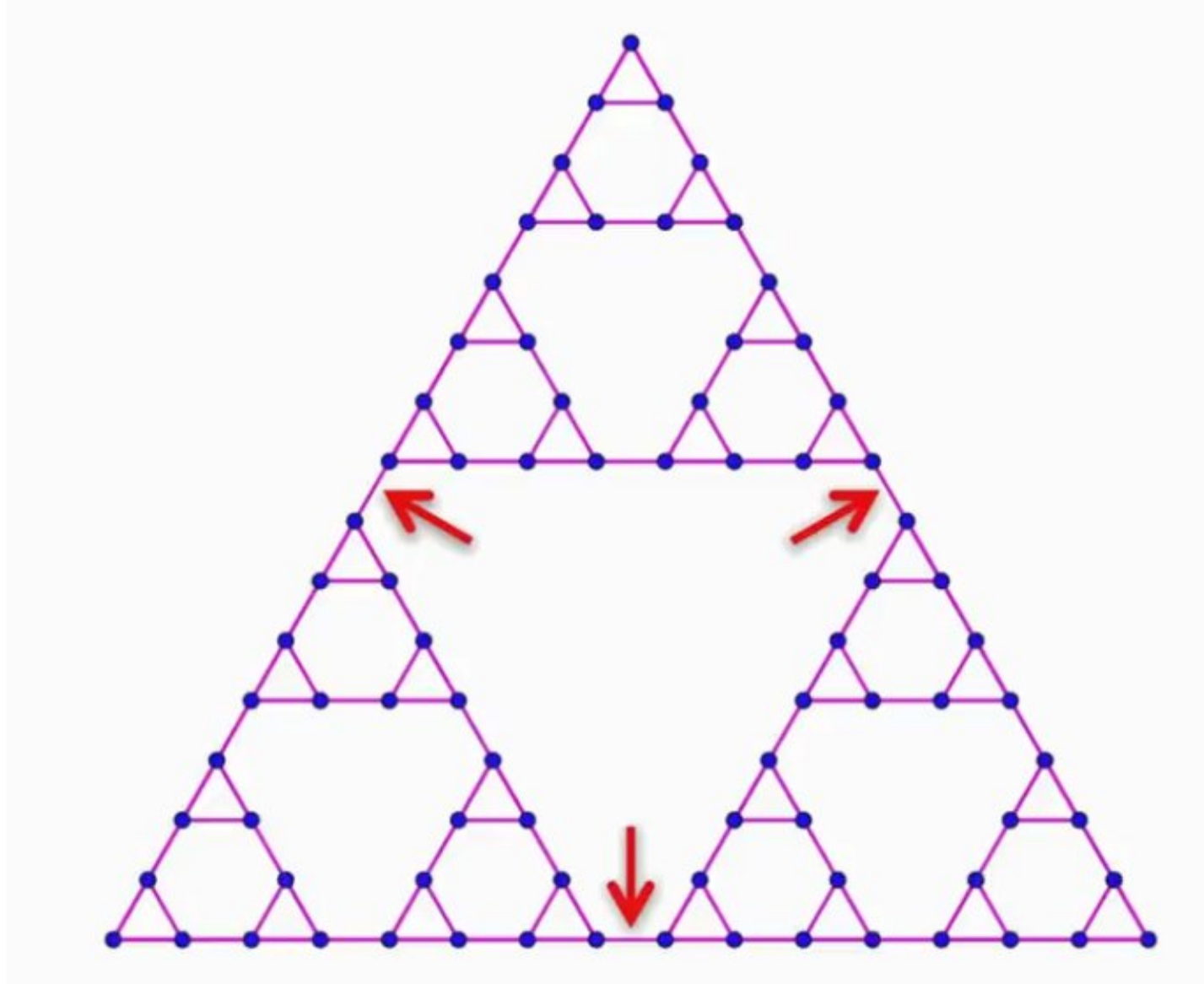
L'algorithme le plus **lent**

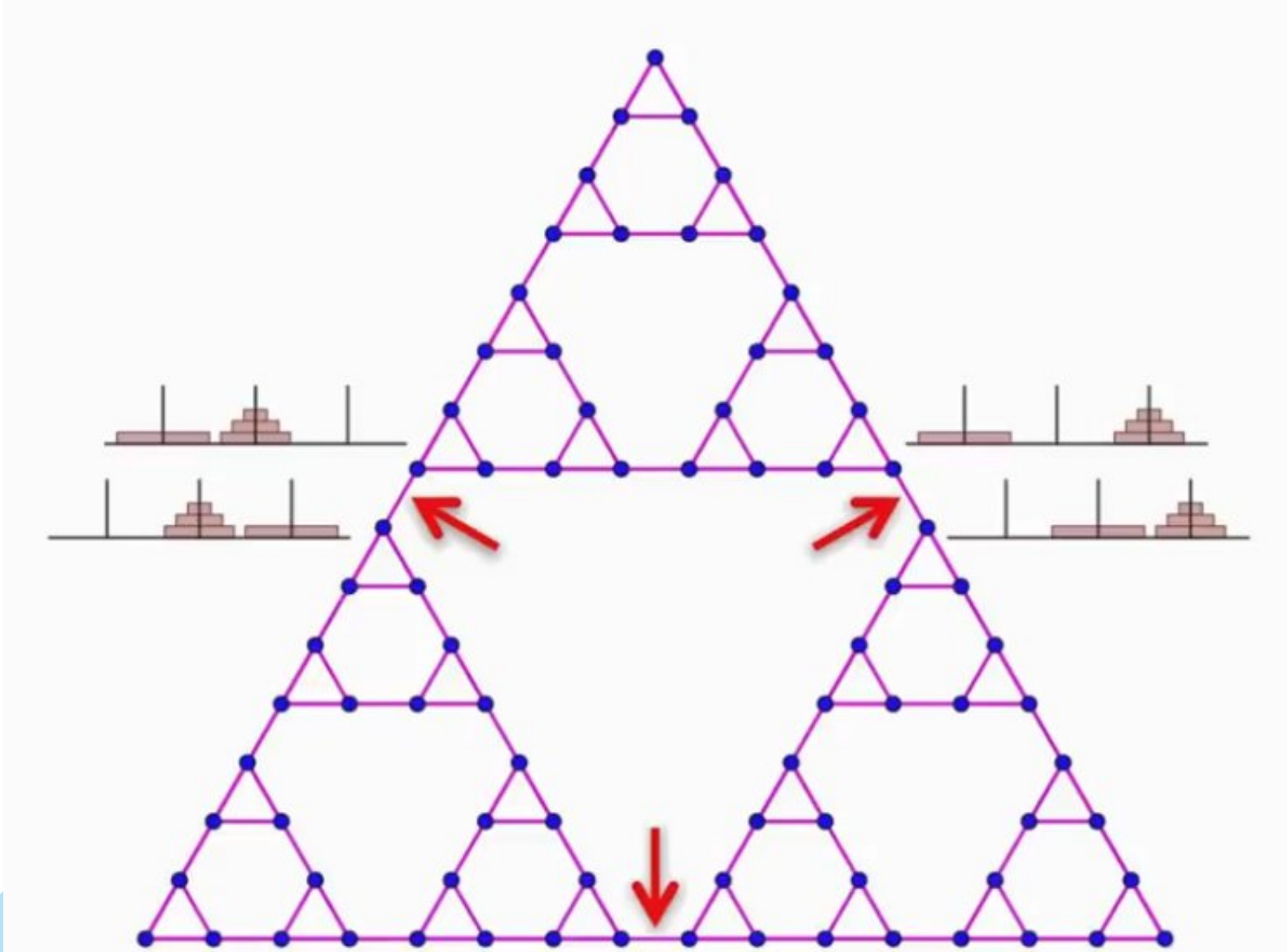
Tant que c'est possible, faire un déplacement qui :

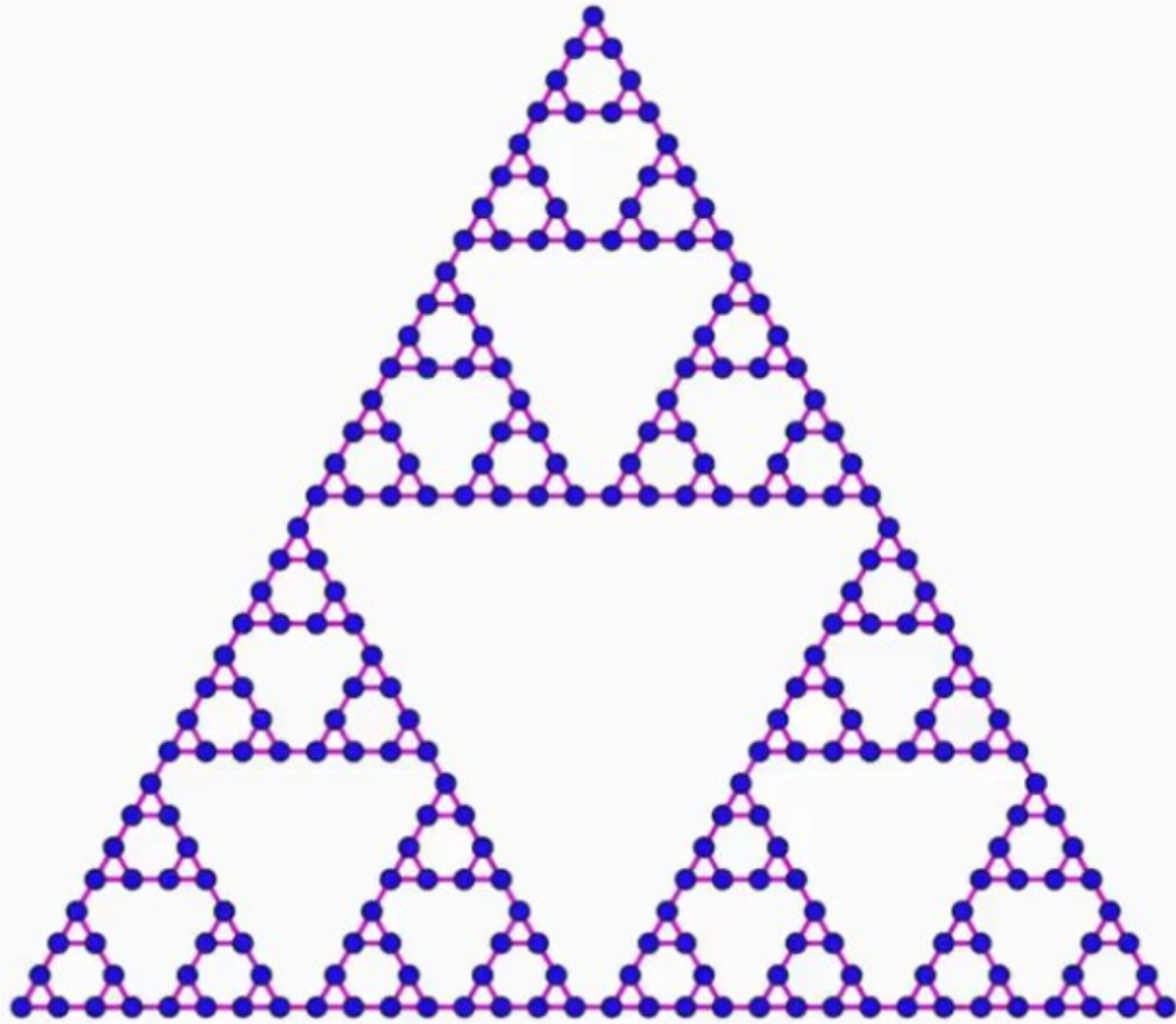
- implique le piquet du milieu
- n'annule pas le déplacement précédent

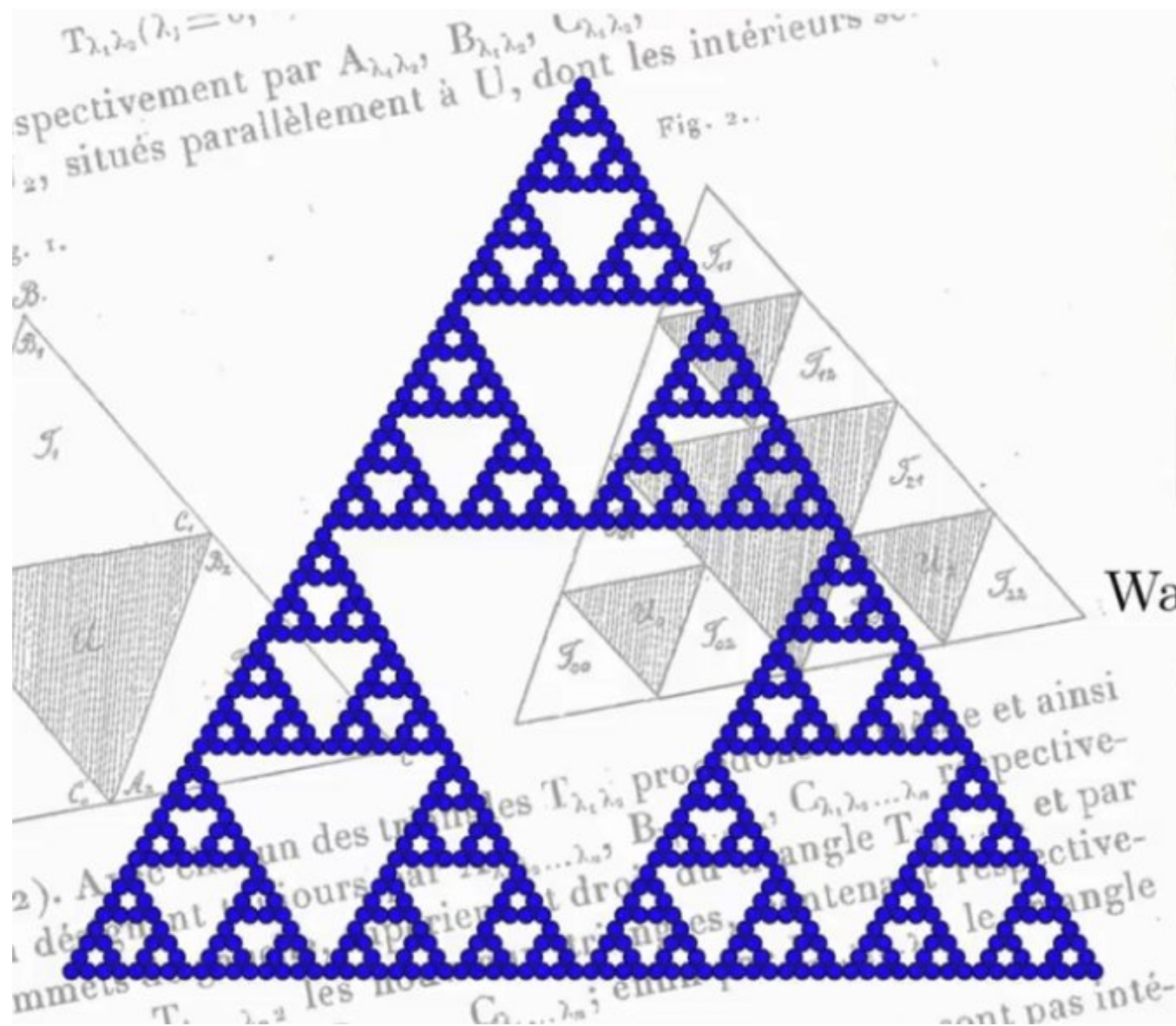












Wacław Sierpiński
 (1882-1969)